“Reaching out to Infinity: The Mathematical Imaginary of Medieval Keralam"

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Abstract: One important intellectual circle which disintegrated and died out after the European invasion of India was the Kerala school of astronomy and mathematics. The significance of Kerala mathematics lies in its rare continuity. According to available information it stretched from the fourteenth to the seventeenth century C. E. The major proponents of the School hailed from Irinjilakuda, Ponnani, Alathoor, Thrikkandiyoor, Vanniyoor and Sukapuram. The generous patronage of the rulers of these regions who promoted studies of advanced nature not only in Sanskrit and literature but also in astrology, and astronomy led to an explosion of knowledge in mathematics. The invention of Calculus and the Infinite series were important milestones in the journey towards the rise of modern mathematics. The present paper examines the advances in the study of mathematics, from finite concepts and ideas to the infinite in Kerala, now widely known as the Kerala School of Mathematics.

Key words: Mathematics, Infinite Series, Medieval Keralam, Astrology, Astronomy

Pyari SuradhKerala is located in the peninsular region of Indian subcontinent which helps in easy access to the place through land and through ocean. The unique geographical features especially the 44 rivers, some of which are so deep and navigable that, in ancient times, were used for inland navigation similar to present day Suez Canal helped the region to develop a constant relation with many places around the globe for trade, tourism, education etc. The geographical location provides Kerala with a warm and moderate climate.
throughout the year which has been very crucial in the development of the place as an agrarian society and also a business centre. The very heavy rainfall received by the region during southwest monsoon and northeast monsoon and the steep terrain helps in constant alluvial deposition, high yield, cultivation in paddy fields and dry land of almost all crops possible, especially high yield of rice. These unique features are at the same time a curse as it causes very drastic and destructive flooding of the region for short periods which loots the land of its wealth. The population in this region always exhibited an interest in all round growth of their society which reflects in the present day also.

As an agrarian society their drive to know about the seasonal and climatic changes accurately can be easily understood. This lead to the intense studies in the fields of mathematics and astronomy. There were also a great number of extended and elaborate work especially translations from earlier works from other parts of India. K V Sarma in his work *History of Kerala School of Hindu Astronomy* reveals the fact that nearly 400 palm leaf manuscripts on astronomy and 350 on astrology have been discovered in Kerala. The conservative mindset of the custodians are the reason for the inaccessibility of this by students and scholars interested in this area. A number of them have perished and the extant ones face threats of perishing without being deciphered in the hot humid climate of Kerala. (Sarma, 1972) In Kerala almost all incidents in a person’s life were celebrated ceremoniously starting from birth which continues even after death. To list a few auspicious ceremonies in a person’s life “vayambu kodukkuka”(feeding the new born a potion made of gold, honey and sweet flag), “choroonu or “annaprashanam (first introduction of solid food and all the tastes to the infant) ,”irupathettukettu” ( the naming ceremony), vidyarambham (initiation to learning), rithukalyanam (celebration of attainment of puberty), “vivaha nishchayam(engagement), vivaham(marriage), pulikoduppu( celebrating pregnancy) ,shradham, (the final tribute giving and adieu to soul). Also horoscope of a person was made for foretelling the incidents in their life. This necessitated the advancement in mathematics, astrology and astronomy.

Apart from these bhoomipooya (ritual in honour of land) kallideel (laying the corner stone for any new building), kattilaveypu (placing of the first front door of a building), grihapravesham (house warming), neetilirakkal( the first introduction of a boat or ship to water), annual festivals in religious centres especially temples. In the field of agriculture
starting from the sowing of the seed, followed by transplanting sapling, harvest, thrashing, winnowing, niraputhari (collecting harvest in barn), harvest festival were all occasions of celebration. And auspicious time were observed for these rituals. This forced them in advanced study of mathematics and astronomy to invent calendars, exact knowledge of summer and winter solstice and season change, start and end of rainy season, period of high tide and low tide etc. for effective agriculture, animal husbandry, fishing and transportation.

Another area is trade, from ancient times Kerala was a well-known port city due to its location, indigenous products including spices, teak and wooden ships known as “uru” which were of great demand. It was accessible to both east, west via sea but the exact time of the winds from land to oceans and oceans to the land now known as westerly or trade winds in the present were crucial in enabling transport of sailing ships via sea. The absence of technologically advanced instruments for tracking sea route and determining directions forced to resort to the knowledge of position of stars which were an indispensible prerequisite in navigation skills. This lead to the advancement in astronomy, mathematics, cartography. Geometry for ship building and also building of houses, religious centres and other great buildings. In order to make accurate predictions a thorough knowledge of the position of stars and planets and seasonal and climatic changes were necessary. And these were the pioneering reason for the birth growth and rapid development of mathematics, astronomy and astrology.

The invention of advanced mathematics in medieval Kerala are a result of the continuity in rich Indian tradition and the relation between advanced civilizations in the past through travel of Indian scholars belonging to all sects including Hindus, Buddhists, Jains, Parsis, Muslims, Christians to educational centres in Baghdad, Chang’an etc. and the diasporic communities in Kerala which includes Jews, Christians, Arabs, Buddhists, Jains etc. The novel ideas thus obtained were incorporated into literature, religion, arts, medicine, science and technology. Some unique works are Charaka Samhita, Sushruta Samhitha, Ashtanga Hridayam, Ashtanga Sangraha, Sharngadhara Samhita, Bhava Prakasha on Ayurveda Maathangaleela on elephantology, Manushyalaya Chandrika on construction of houses (extant version of both authored by Thirumungalath Neelan Musath and older with a commentary by Paloli Choi Vaidyar) etc. All of them give us a knowledge in the traditional mathematics in the form of weights and measures, flawless construction manuals etc. These
works were found as written in Sanskrit, Manipravala (combination of Sanskrit and Tamil – believed to be the ancestor language of Malayalam) and in Malayalam.

Though the Sanskrit tradition, that is learning and teaching of Sanskrit were enjoyed only by the Brahmins, the vocational pursuits and scientific and technical knowledge were known and practised by various communities. Astrological predictions and calculation of auspicious time were done by both Brahmin and Ganaka community. Members of Ganaka community also practised medicine. Medical practitioners also belonged to the communities of Brahmin, Ezhava, Vela or Mannan, Nair etc. Hendrik van Rheede expresses his indebtedness, in the preface to *Hortus Malabaricus*, towards Itti Achuden, who was a Thiyya, along with other Brahmin physicians. The master craftsman or master carpenter was an honorific title which refers to a legendary carpenter, Raman, whose parents were Vishwakarma. The famous temple architectures which were believed to be designed and built by Raman Perumthachan were numerous. Some of them are Shiv temple at Uliyannoor, Chengannur Mahadeva temple, Vazhapally Siva temple.

The legendary astronomer, Vararuchi, is believed to be the pioneer of Kerala astronomy and the author of *Chandravakyas* (Moon Sentences – mnemonic words or phrases which is a set of 248 numbers to calculate the longitudes or positions at different intervals of time to help worshippers carry out their daily observances and rituals). Chandravakyas were also used in Tamil Nadu to construct almanacs. It is estimated that Vararuchi died during fourth century BC.

The unique systems of numeration Katapayadi and the Parahita system for performing calculations in mathematics and astronomy were a refinement of earlier Aryabhata’s system. Bhuta Samkhya is the older system and is of a connotative type. Bhuta the term means element or part or component etc and samkhya is number. Here numbers are indicated by familiar concepts or objects having the same number of parts as the indicated or intended number which is commonly understood in the community in which it is used. Some commonly used denotations were:-

0 denoted by randhra (hole)/ sunya (void)/ purna (whole)/ kha (sky)/ antariksa (atmosphere) etc

1 denoted by bhumi (earth)/ sasi (moon)/ go (cow) etc.
2 is denoted by netra (eyes)/ bahu (hands)/ karna (ears)/ paksha (moon’s waning and waxing periods) etc – each being a pair.

3 is denoted by kala (time - past, present and future), loka (heaven, earth, and hell) etc- each of which is a set of three components.

4 is denoted by disa or dik (directions-east, west, north, south), yuga (krta, treata, dvapara, and kali), veda (rig, yajur, sama, atharva) etc- each of which is a group of four.

5 is denoted by pandavas, bhuta (elements) which have five members.

12 is denoted by mas (months), rasi (signs of the zodiac) – groups having 12 constituents

32 denoted by danta (teeth)

Synonyms of these words also represent these numbers. It employed the principle of place value and mode of writing or representation was from left to right .Details regarding the origin, inventors and first practitioners are unavailable.” (Parameswaran, 14) The limitations of this system is that only a person who is familiar with the concepts like pancha pandavas (pandavas who were sons of pandu and 5 in number), saptharishis (the seven holy sages of Hindu mythology) will be able to decode or decipher these numbers .Another challenge was the ambiguities in some of the representative concepts like paksha which is a partition of a month into two and included 15 days so it sometimes stood for both 2 and 15; concepts like loka meant 3 and also 14; dik or disa for 4 and 8 which could have been north, south, east, west or north, north east, north west, south, south east, south west, east and west. It makes the deciphering of Bhuta Sankhya prone to mistakes and affected its prolonged use and survival. Katapayadi system overcomes this defect.

The system was used by astronomers and mathematicians of India because it enabled the construction of rhythmic shlokas or poetic or rhythmic passages which could be memorised easily by them and their posterity and transferred through rot learning and oral method in those periods. The immense number of synonyms to choose among to write the same number in variety of ways was fruitful in providing a poetic language to scientific and mathematical formulae and texts which is a unique feature of Indian system. According to Datta and Singh “a system of expressing numbers by means of words arranged as in the place-value notation was developed and perfected in India in the early centuries of the Christian
era.” (Parameswaran, 15) At the beginning Sanskrit alphabets were used to denote this system of numeration but in due course Malayalam language was also used in this system because all sounds are similar in pronunciation in Sanskrit and Malayalam only the script, that is notation is different. The Katapayadi system derives its name from k, t, p and y the first letters in the respective groups of consonants. Katapayadi system follows the decimal notation in which the number at unit place is written first, tenth place written next and so on from right to left. The letters (k, ṭ, p, y) represent 1, (kh, ṭh, ph, r) represent 2 , (g, ḍ, b, l) represent 3, (gh, ḍh, bh, v) represent 4, (ṅ, ṇ, m, ŝ) represent 5, (c, t, ṣ) represent 6, (ch, th, s) represent 7, (j, d, h) represent 8, (jh, dh, ḍ) represent 9, (ṅ, n) represent 0 respectively. Apart from ṇ, ṇ, all pure vowels if not preceded by a consonant represents 0. Consonants followed by a vowel represent the same number as the consonant without a vowel so p, pa, pi, pu denotes one only. The popularity and advantage of this representation rests on the fact that numbers having several digits can be put down as meaningful expressions and several different expression for the same number is also possible. The mnemonic of a number is named as its paralperu. A variant of Katapayadi was used by Aryabhatta II. The Kerala model of Katapayadi was believed to have spread to entire south India. (Datta, 69) Kerala had a unique set of symbols to represent the numbers from 1 to 10. These symbols were commonly used in calenders and calculations in native schools.

Kerala also followed traditional indigenous divisions of an year which goes as 1aandu or kollam=2 ayanam=6 rithu=12 masam=24 paksham ≈ 52 aazhcha=365 divasam. And division of time as 1day=8 yamam=60 nazhika=3600 vinazhika=21600 asu or pranan=216000 gurvaksharam. Traditional measurement of solids like volume of grains were done by using vessels made out of wood or metal some measurements are, one naazhi =14400 rice grain, 3 naazhi = 1 chanthiram, 1idangazhi=4 naazhi , 1.5 idangahi= 1thooni, 1 para = 12 idangazhi. Commodities like gold, silver, medicines were measured using kazhanjikol, also kazhanji seed were used to measure small weights 1 kazhanj =20 lucky red seeds,1 thola equivalent to 11 gm, 2 thola = 1 azhakku, vellikkol were also used. Liquids (oil, milk) were measured using thudam, 4 thudam = 1 nazhi, 1 kutti = 5 nazhi, length measured by wooden scale called muzhakkol, 1 muzhakkol = present day 30 inch. So Kerala had a unique system of measurement which shows a fully independent society Apart from names for numbers from one to nine there are multiples of ten up to ninety and Indians used separate names for the powers of ten up to 1017:-
Eka=1, dvi=2, tri=3, catur=4, pancha=5, sat=6, sapta=7, asta=8, nava=9, dasa=10, vimsati=20, trimsat=30, catvarimsat=40, panchsat=50, sati=60, sapta=70, asti=80, navati=90, sati=102=100, sahasra=103=1000, ayuta=104=10000, niyuta or laksha=105=100000, prayuta=106=1000000, koti=107=10000000, arbuda=108=100000000, abja or vrinda=109=1000000000, kharva=110=10000000000, nikhara=111=100000000000, mahapadma=112=1000000000000, sanku=113=100000000000000, jaladhi=114=1000000000000000, antya=115=1000000000000000000, madhya=116=1000000000000000000000. So Indians may have taken 10 as the base. In the forming of words of multiples of ten the presence of multiplicative principles is seen. And for expressing larger numbers the additive method was used. The subtractive principle also was used in some cases.

It was during the reign of Ravi Varma Kulasekhara, the Chera king a new era was launched. It was in middle of August of 825 A.D, the era was named kollavarsham. This is being mentioned to elucidate the fact that mathematical and astrological advancement in Kerala were at an advanced and propitious stage more than seven centuries before Kepler (1571-1630 A. D.) who enunciated laws of planetary motion. "Science does not bloom suddenly in a vacuum, but since knowledge is cumulative, growing bit by bit through generations and centuries, it needs a firm foundation of rationalism, openness of the mind, strict self-discipline, leisure, hardworking habits and continuity of culture for its sustenance. It is quite surprising that it was at a time when originality and creativity has almost dried up in other parts of India, that Kerala produced a number of great works in these fields which placed her scientists far ahead of scientists everywhere else including Europe."(Narayanan,11)

According to K Kunjunni Raja the presence of a large number of wealthy kingdoms were the reason for rich contributions by scientists, astrologists, astronomers, mathematicians, poets, artists etc. The scholars never felt any political barriers within Kerala they were welcomed everywhere and there was a healthy competition to provide patronage to scholars. Though Kerala was politically divided into small kingdoms they were culturally united and prosperous. This welcoming atmosphere attracted scholars even from Tamilnadu to mingle with scholars of Kerala and spent a considerable part of their life here exchanging ideas and thoughts and translating works, engaging in fruitful debates. The constant income from trade through land and sea, the constant contact with various diasporas provided for the stability
in Kerala economy when other parts of India were being attacked by Muslim rulers who even came to Kanyakumari, Kerala was left undisturbed due to the natural shield Western Ghats. Late medieval Kerala scholars attempted to reform calendar using Parahitaganita and Dr̥ggaṅita theories using information obtained by direct observation and spherical geometry because of the lack of any instruments for advanced sky watching.

The stalwarts in the field of astronomy and mathematics in Kerala start from the pioneer Vararuchi. His only extant work is Vararucivākya (Candravākyāni). Haridatta (650-700 A.D) in his work Grahaḥāranibandhana used Katapayadi system which was known by another name until 9th century. "He made corrections in Āryabhaṭa system and invented the Parahita system of astronomical computations. It was presented at the Māmāṅkam festival of 683 A.D at Thirunnavaya. He also introduced a smaller Yuga called Dhījaṅgannupura- yuga, of 576 years or 210389 days (which is 1/7500th part of a Mahayuga) and accurately determined the zero corrections for this subyuga for the mean motion of planets. The corrections are used to compute the mean planets for any given date.” (Anil Narayanan, 206). Haridatta refered to his own another work named Mahāmārganibandhana in Grahaḥāranibandhana which is yet to be discovered.

From the age of the Perumals of Mahodayapuram (9th to 12th centuries) to the period of swarupam organisations (13th to 17th centuries) is generally known as the medieval period in Kerala history. Kerala School of Mathematics and Astronomy flourished during this period. Govindasvāmin (800- 850 A.D) was the court astronomer of King Sthana Ravi Varma Kerala. He was one one of the ablest exponents of Bhaskara I and the Aryabhatan system. His elaborate bhashya or exegesis on Mahabhaskariya namely Mahābhāskariyabhāṣya contains new ideas and mathematical elaborations which are yet to be fully recognised and studied from the point of modern mathematics. An original work Govindakriti which has been quoted by later writers like Sankaranarayana, Nilakantha is yet to be recovered. Govindapaddhati a work referred to Govindaswamin by Nilakantha and Sankaranarayana and manuscripts of short commentary called Prakatārtha or Sampradāyapradipika on Parāsarahora are mentioned by Kunjunni Raja in The Contribution of Kerala to Sanskrit Literature.

Sankaranarayana(825-900 A.D) was a native of Kollapuri on the Arabian coast which could be most probably present day Kollam. He was the chief court astronomer of Sthanu Ravi
Varman, the 9th century ruler of the Chera Dynasty. He was a disciple of Govindaswamin. “He has given the standard mathematical methods of Āryabhata I such as the solution of the indeterminate equation by $= ax \pm c$ (where $a$, $b$, $c$ are integers) in integers. This was known as kuttākara method for the determination of mati, which refers to the optional number in a guessed solution, which is a feature which differs from the original method presented by Bhaskara I.”(Anil Narayanan, 208). His only known work is a commentary on the Lakhubhaskariya named Laghubhaskariyabhasya which was written in 869 A.D. It is a highly elucidatory work with regard to the subject discussed. It shows the keen interest of the royalty in promoting the study of astronomy. The Perumals of Mahodayapuram in their capital city of Mahodayapuram built an astronomical observatory which could have been roughly somewhere in modern Kodungalloor near Cochin. Sankaranarayana refers to the observatory and the practice of public announcement of correct time at regular intervals.” The kind of an intellectual atmosphere that prevailed at the time may be gathered from references in the Kokasandesham, a 15th century text written in Manipravalam. Kokasandesam refers to a great teacher (or preceptor) and his students (sisya) at Triprangode.” (Vijalekshmy. M, 61)

Another scholar is Suryadeva Yajvan (1191-1250 A.D) belonged to the Nidhruva gotra and nephew of another Suryadeva. He was a versatile commentator and very much recognised authority in the subject. His extant works include elaborate commentaries on Āryabhatiya and Laghumanasa. On astrology he has written a commentary on Mahayatra of Varahamihira and Jathakakarmapaddhati, both of which were famous works in the concerned field. He has given his date of birth as visvesa (1113) saka in one of his commentaries.

It is very important to note that Gurukula system of education prevailed during this time. After the initiation ceremony early or primary education is given and after that a student is accepted by a guru as a sishya, then the student becomes a part of guru’s family. The instructions of the guru would then be followed and the student is supposed to serve and obey the guru. During the learning process the teacher will orally instruct and make the sishya proficient in the instructed things through rote learning and periodical chanting, the content may be noted down on palm leaves. The student will be taken to participate and listen to various discussion in parishads where great masters discussed and presented their
knowledge and findings. There were also exposure to vidvat sadas where scholars took part in debates. The main gurukulas during medieval period in Kerala were Trichur Brahmashwam Matham, Thirunavaya Samuha Matham, Kudallur Mana Gurukulam, Thiruvalla sala, Moozhikulam sala, Kudungalloor and Punnasseri Gurukulam. The intellectual liveliness of the period is also evident from literary competitions like Revathipattathanam at Kozhikkode and Kadavallur Anyonyam at Kadavallur (near Kunnamkulam).

The emergence of major figures of the Kerala School occurred between 14th and 15th centuries. Madhava of Sangamagrama (1340-1445A.D) was referred as Golavid (Master of Spherics) by later astronomers. He was an astute mathematician who came from the medieval Brahmin settlement namely Sangamagrama. Sangamagrama a village with a temple dedicated to a deity of the same name is identified as present day Irinjalakuda near Cochin. He belonged to a priestly class, the Emprantiri, consisting of Brahmans who arrived from coastal Karnataka and later became a sub-caste of Kerala Brahmans. The information regarding the name of his family house and village were obtained from details provided in his own work Venvāroha, in its commentary by Achyuta Pisharati and in the Āryabhatiyabhasya of Nilakantha. It is understood that his house name meant that in the compound stood an Ilanji tree in Malayalam or Bakula tree in Sanskrit that is because the house was called ‘Bakuladhistitaviharam’ or Ilanjipalli. In present day Irinjalakuda there exists two nambuthiri houses with names similar to this namely ‘Iringarapalli’ and ‘Iriaravalli located about eight kilometres from exact Irinjalakuda near Kallettumkara railway station. The temple in which Madhava is believed to have spent many hours meditating is now under the management of Iringarapalli Mana.

Madhava in his works Venvaroha and Sphutachandrapati, revises and refines the Chandra vakya system of Vararuchi. He calculated the exact positions correct to the second, and evolved the procedure to find out exact position of the moon every 36 minutes. Vararuchi gave values correct to the minute. Considering the cyclic nature of lunar vakyas where nine months is equal to 248 days he found the lunar longitude at nine equally distant times in one day. Computation of the longitudes of planets were also discussed. Short commentaries are available on his works Mahājyānayanaprakāra and Mahdyamānayanaprakāra. These works contain novel theorems and computational methods found or evolved by him and used later by his successors in the field. Kunjunni Raja gives the name of another important work
Aganita. Aganitapancanga is also attributed to him. An anonymous work Aganitagrahacara which is quoted in Karanapaddhati is available in manuscript which mentions sodhyabdas, Lagnaprakarana, Golavada are also attributed to him. “In the case of Aganitagrahacara in, the sodhyabdas, with A. D. 1418 as the latest among them, agrees, surprisingly with the date of Madhava. This goes to confirm that in the present Aganitagrahacara we have a hitherto unidentified work of Madhava, viz. his Aganitapancanga which is mentioned in the said astronomical document” (Kunjunni Raja 52). Kunjunni Raja also hints the possibility of comprehensive treatise on astronomy by Madhava which is yet to be discovered.

It is possible to be true as M R Raghava Varier as part of an investigative research found that many uncatalogued and catalogued books from India were present in Bibliotec Apostelica Vaticana and Nazionale Centrale di Roma in Rome. This may be only the tip of an iceberg. India was a hub of traders, travellers, tourists, colonizers what is yet to be discovered from individual collections, archives, libraries around the world could leave us awe inspired. The interesting fact is that they are mainly manuscripts in languages native to India which includes Sanskrit, Malayalam, Manipravalam, Tamil etc. So it is possible that these were not collected in vain just for sake of collection or as a hobby. Foreigners from many regions who came to India made deliberate attempts to study Indian languages, it was not just out of curiosity. When they came into contact with a rich culture it was obvious that there were possibilities to acquire higher levels of knowledge which they imbibed and articulated for the modernization of their societies.

Paramesvara of Vatasreni (1360-1455) was Madhavas disciple and another major member of what is now being identified as Kerala School. He revised the Parahita system of computation and introduced Draganitha in the year 1430. He hailed from the village Ālattur (Asvatthagrama). His house was situated on the banks of Nila (Bharathapuzha) near its confluence with Arabian Sea. He is reputed to have carried out observation of sky, eclipses and investigated for fifty five years and his findings are documented in Siddhantadipika. Paramesvara was a prolific writer who authored about 30 works which include original treatises as well as commentaries. His notable compositions Draganitha(1430), Goladipika I-III(1443), three works on improving computations and explaining the rationale of eclipses namely Grahanastaka, Grahanamandana and Grahananyayadipika. Grahanamandana discusses the method for the construction of the eclipse graph for any desired moment.
Grahanastaka deals with the calculation of eclipses since much matter had to be compressed in the eight verses the work is very terse and technical and an apt example of the Indian mode of intellectual patentship and the tradition of representing only the crux of the findings which they were sure their posterities could decipher and which could be sustained through the gurukula system. Candracchayaganita, a text on the computation of moon shadow, Vakyakarana, on the computation of mnemonic tables. He has written commentaries like Āryabhatiya, Karmadipika on Mahabhaskariya, Siddhantadipika Bhāsya on Mahabhaskariyabhasya, Paramesvari on Laghubhaskariya and Lilavati, commentary of Laghumanasa of Munjala, commentary on Suryasiddhanta and also wrote a gloss on his own work Goladipika and Vyatipataastaka. Two of his works on astronomy namely Vakyadipika and Bhadipika are yet to be recovered.

Damodara (1410-1510) next in the line was son of Paramesvara of Vatasreni, no fulfledged work of his is known till date. His pupil Nilakantha Somayaji refers to him as an erudite astronomer and scholar and quotes from his writings. Later writers also mention Damodara and Nilakantha quotes Damodara which suggest that he might have composed certain works which are yet to be discovered.

Nilakantha Somayaji (1444-1545) the centenarian astronomer who hailed from Trikkantiyur (Kundapura), near Tirur in South Malabar. He was the student of both Paramesvara of Vatasreni (received occasional instruction) and his son Damodara of Vatasreni. Nilakantha also had another teacher Ravi who authored Ācaradarsana. During his student days he stayed at their house as part of gurukula system. In his Bhashya on the Āryabhatiya, Ganitapada he gives full details about himself. In his work Siddhantadarpāna-vyakhya he gives his date of birth as ‘tyajāmyajnatām tarkaih (16, 60, 181) which falls in Dec. 1444. He and his brother Sankara were patronized by the hereditary religious head of Namputhiris, Kausitaki Adhya Netranarayana (Āzhvānceri Tamprākkal). He was a prolific writer and erudite scholar. The mathematical sections in his work Tanthrasangraha (1500 A.D) helps scholars for a better understanding of Madhava’s contributions because he was the first person to record and elaborate the findings of Madhava. It also formed the basis of Yuktibhasa. In his work Golasara he discusses spherical geometry and his view on astronomical topics. In Siddhantadarpāna he puts forth a set of astronomical constants (32 versus). Āryabhatiyabhāsya is considered to be his masterpiece through its extensive commentary.
Valuable quotations. *Jyotirmimamsa* stresses the necessity of revising and correcting data obtained through observation. *SundararajaPrasnottara* are a set of answers for astronomical problems addressing a Tamil scholar who commented on the Vakyakarana of Vararuchi. *Grahananirnaya* and *Candrachayaganita*, also deal with astronomy. He also wrote a malayalam commentary on *Candrachayaganita*. *Grahaparisakrama* provides the procedures for the observations planets and methods of their computation, it is a set of 200 verses.

Chitrabhanu Namputiri (1475-1550) hails from Sivapuram near Trichur. He authored an advanced work in four chapters regarding astronomical computation titled *Karanāmrta*.” Chitrabhanu gave integer solutions to 21 types of systems of two simultaneous Diophantine equations in two unknowns.”(Anil Narayanan 221) He was Nilkantha Somayaji’s student, whom he addresses as Gargya in his work. The date of composition of *Karanamrta* given in Kali chronogram is 1530 A.D.

Trkkutaveli Sankara Variyar (1500-1560) lived at Trikkutaveli near modern Ottapalam. He was a disciple of Nilakantha Somayaji. He wrote an elaborate commentary on *Tantrasangraha* titled *Laghuvivrti* in 1556 A.D. The name of the author was not mentioned in the commentary, the ‘post-colophonic’ statement by the scribe in Malayalam in some manuscripts:” i vyakhyana Trkkutaveli-c-Cankaravariyar otukattu camaccatu / Āzhvancerikku ventittu sukhamse siksiccu camacu ennu Parannottu parannu kettu / : ‘This commentary was composed last by Trkkutaveli Sankara Variyar. It is stated to have been said by Parannottu that it was composed with great care for the sake of Āzhvanceri’.”(Kunjunni Raja 58) The person who is referred to as Parannottu is probably Parannottu Jyesthadeva, a young contemporary of the author. *Kriyakramakari* which is a commentary on Bhaskara II’s *Lilavati* was begun by him and written upto 199th verse but was completed by Narayana an esteemed associate of Nilakantha. The work *Kriyakramakari* has a unique and revered place in the history of Kerala mathematics and astronomy because of the detailing on earlier works and authors some of them are extinct and this book stands as an evidence to their existence. Other works attributed to him are *Kriyākalāpa*, *Karanaśāra* (a work in four chapters) and *Karanaśārakriyākarma*, malayalam commentary on the work *Karanaśāra*. 
Jyesthadeva (1500-1610) is the well known author of the popular work Yuktibhasa or Ganitanyayasangraha. He was a student of Damodara of Varasreni. It is believed that Nilakantha was his senior colleague and a respected elder. He lived in Ālattur village and was a member of Parannottu family of South Malabar. The first part of the detailed exegesis is an elaboration of the rationale of mathematics and second part of astronomy. The Sanskrit version is probably his work and is named Ganitayuktibhasa. The authorship of Drkkarana, available in single manuscript, metrical treatise on astronomy written in Malayalam is attributed to him.

Acyuta Pisārati (1550-1621) was a great scholar in astronomy, medicine, grammar, poetics who lived in Trkkantiyur in Tirur (Kundapura in South Malabar). He discovered ‘reduction of the ecliptic’ and stated it in Sphutanirnaya which was elaborated in Rāsigolasphutaniti. This correction was introduced in western astronomy also at about the same time by Tycho Brahe. He was the teacher of Melpathur Narayana Bhatta and protege of King Ravi Varma of Vettathunadu. The date of Achyuta Pisarati’s death is given in the ‘carama –sloka’ composed by Melpathur Narayana Bhatta as ‘vidyatama avar asarpat’ which falls in 1621 A.D. “Rāsigolasphutaniti in fifty verses, discusses a very prominent point in celestial measurements, that is whether the longitudes of planets as well as stars are to be measured along with ecliptic or each along its own path- concerned with the reduction of the moon’s true longitude in its own orbit to the ecliptic.”(Anil Narayanan, 220). He has written a Malayalam commentary Venvarohavyakhya on Venvāroha of Madhava of Sangamagrama and it was written at the request of Netranārāyana. Acyuta has also authored several works including Karanottama on astrological computation of the mean and true longitudes of planets, Uparagakriyakrama, on solar and lunar eclipses, Uparagavimsati and Chayastaka on eclipse and shadow computation respectively.

Some scholars continue with the list of Kerala mathematicians and astronomers with successors of Achyuta Pisarati and their respective students. The chain continues with Trippanikara Potuval, Melpputtur Narayana Bhattatiri, Navayikkulam Azati, Pulimukhattu Potti, Raman Āsān, Krishnan Āsān, Mangalattu Dakshinamurtti Musatu,Mannar Nalekkalattil Balarama Pilla, which only comes to a stop with Kilimanur Karindran Tampuran(1812- 1846). The continuity of a tradition lies in the dedication of the posterity
in sustaining the enthusiasm and empirical attitude and the role of the Kerala mathematician in an age without printing, external thrust is commendable.

As there is no record of any original contribution after Acyuta Pisarati it is better to include the members till him for the convenience of the study. But a fact that requires mentioning is that only a small fraction of these works have been subjected to study because some were not able to withstand the ravages of time. Many manuscripts still are under the tag of artefacts with obscure script of old Malayalam or crude forms of Sanskrit and have not been subjected to rigorous study. The present knowledge available is from only a handful of texts. There are hundreds of texts waiting to be studied in manuscript form. Europe is rectifying its curriculum from school level incorporating Sanskrit and Indian knowledge realizing the abundancy of knowledge awaiting in Sanskrit texts, India is abandoning the language and the rich corpus of knowledge awaiting in it labelling it as outdated. The spirit of enquiry exhibited by the scholars of Kerala school has been lost by their successors and our generation finds it relaxing and authentic just to work on the knowledge that reached India from western mode of education.